§ 17. A Simple Systematic Method to Treat Diffusion Processes Due to Given Electromagnetic Fluctuations

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In given electromagnetic fluctuations, there may be cases that the transport of test particles is regarded as a diffusion process due to stochastic instability of orbits. Up to now, the diffusions due to electric fluctuations and magnetic fluctuations were considered separately. There is no systematic way to treat such diffusions in a same framework. Meaningful is to develop such a simple systematic method that treats the diffusion under the situations electric and magnetic fluctuations coexist, because such the treatment and the results make the most fundamental basis of the transport, when we interprete the complex results of self-consistent electronmagnetic simulations.

A simple systematic method is based on the thought that the deterministic equations of orbits is regarded as stochastic differential equations due to the stochastic instability of particle orbits:

$$\begin{aligned} \dot{x} &= h(x_u(t),t)+\tilde{g}(x(t),t), \\ x_u(t) &= \text{unperturbed orbit}, \\ \tilde{g}(x(t),t) &= \text{stochastic part} \end{aligned}$$

Thus, parts of the original equations described by using the electromagnetic fluctuations are treated as stochastic fluctuations. A main assumption is that the statistical properties of such stochastic parts is Gaussian:

$$\begin{aligned} C_1(t) &= \langle \tilde{g}(x(t),t)\rangle = 0, \\ C_2(t) &= \langle (\tilde{g}(x(t),t))^2\rangle, \\ C_{n\geq 3}(t) &= 0, \\ \mathcal{R}(t,\tau) &= \langle \tilde{g}(x(t),t)\tilde{g}(x(\tau),\tau)\rangle, \\ C_2(t) &= \mathcal{R}(t,t). \end{aligned}$$

By solving the stochastic differential equations, a nonlinear equation on the Lagrangian autocorrelation fucntion of the velocity is obtained, which corresponds to a renormalization:

$$\mathcal{R}(t,\tau) = F\left[\mathcal{R}(t,\tau)\right], \quad F \text{ Functional}$$

The diffusion coefficient is related to a time integration of the Lagrangian autocorrelation fucntio:

$$D(t,t_0) = \int_{t_0}^{t} d\tau \mathcal{R}(t,\tau)$$

Since we are interested in a long term behaviour, an approxiamted solution with a decay form is pursued for the nonlinear equation:

$$\mathcal{R}(t,\tau) \propto \exp[-(t-\tau)/\tau_{dc}]$$

The decorrelation time of the solution τ_{dc} is determined by the diffusion coefficient itself. In a final step of the procedure, the summation of the wave numbers is replaced by the integration as is pointed out by Krommes in order to average out fine structures:

$$\begin{aligned} D &\propto \sum_{k_{||}} \tilde{g}^2_{k_{||}} H(x_0,D,k_{||}) \\ \sum_{k_{||}} &= \frac{1}{\Delta k_{||}}\int_{k_{||min}}^{k_{||max}} \\ D &\propto \frac{1}{\Delta k_{||}}\int_{k_{||min}}^{k_{||max}} \tilde{g}^2_{k_{||}} H(x_0,D,k_{||}) \end{aligned}$$

The resultant diffusion coefficient has simple analytic forms in various limits, from which the differences of the diffusion by electric fluctuations and magnetic fluctuations are clarified. The differences by particle species are, also, clear:

$$\hat{\omega}_m \equiv \left\langle \omega_{mk_{||}} \right\rangle_{k_{||}} - m\omega_{E\times B},$$

$$\begin{aligned} D_r \sim\ & \frac{L_{||}}{4\pi}\sum_m \left\langle v_{||}\left[\frac{\delta B_{rmk_{||}}}{B}\right]^2 + \frac{1}{v_{||}}\left[\frac{\delta E_{\theta m k_{||}}}{B}\right]^2 \right\rangle_{k_{||}} \\ & \times \left\{ \mathrm{Tan}^{-1}\left[\frac{\delta k_{||}v_{||}-\hat{\omega}_m}{\overline{k}_r^2 D_r}\right] \right. \\ & \left. + \mathrm{Tan}^{-1}\left[\frac{\delta k_{||}v_{||}+\hat{\omega}_m}{\overline{k}_r^2 D_r}\right] \right\} \end{aligned}$$